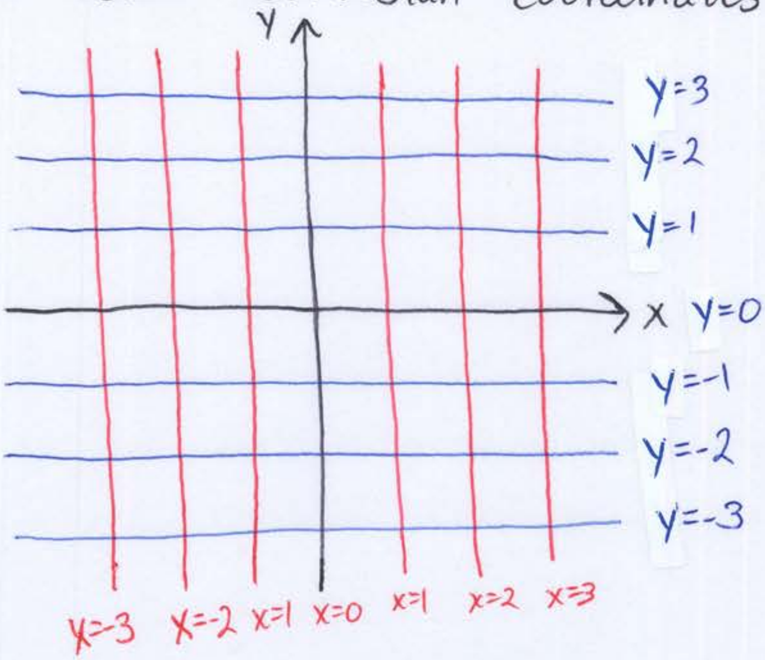
Lecture 34

10.3 - Polar Coordinates

Coordinates on the plane $\mathbb{R}^2$ are basically a choice of "grid lines" on the plane (grid lines are the graphs of equations "coordinate = constant"). For example, the grid lines for the usual Cartesian coordinates looks like:

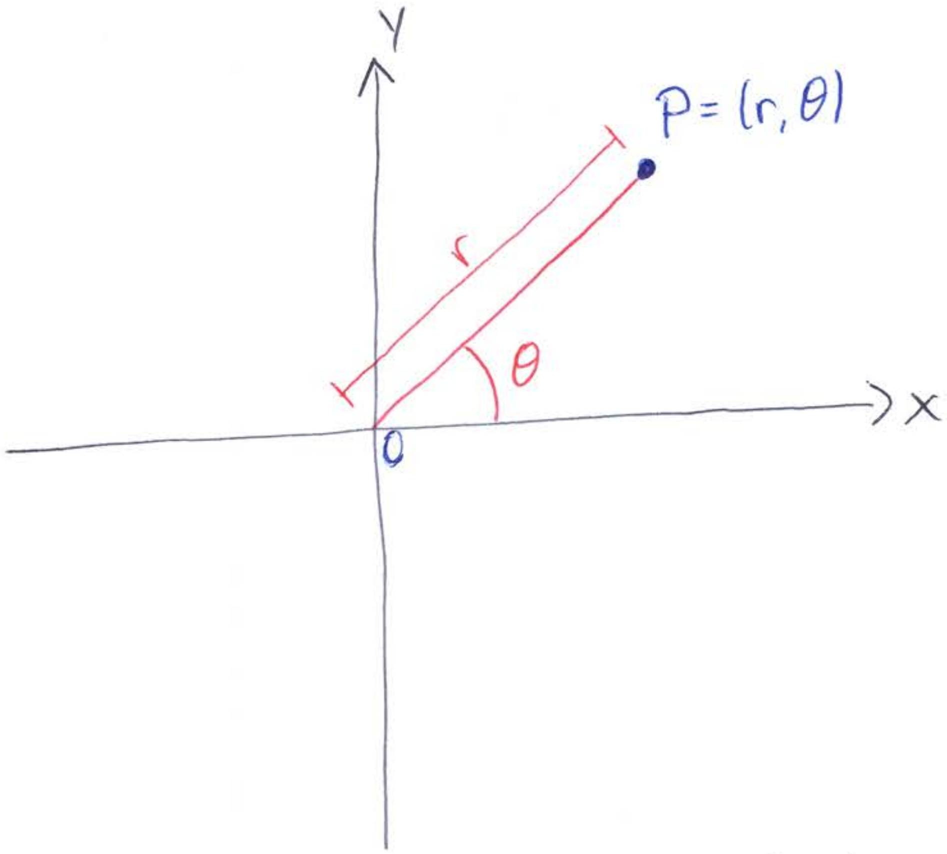


Polar Coordinates aims to describe the plane in terms of circles, centered at the origin. This gives 2 coordinates

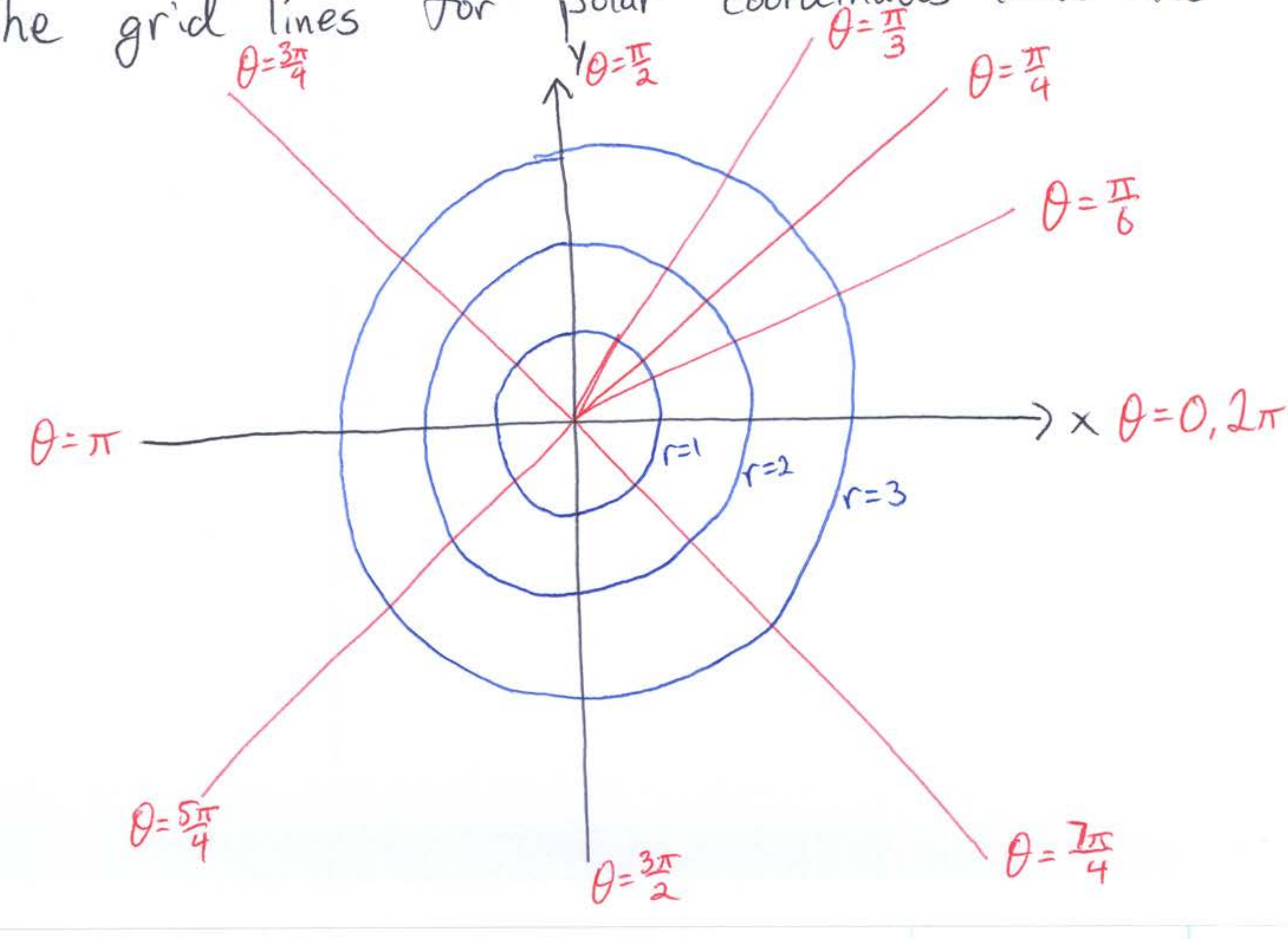
1) distance from the origin $= r$

2) angle of the ray connecting the point to the origin with the positive x-axis $= \theta$

Graphically, these points look like:



The grid lines for polar coordinates look like:



Notice that the graphs of:

- $r = c$ is a circle of radius c, centered at the origin
- $\theta = \alpha$ is a ray (half-line) starting at the origin and making an angle α with the positive x-axis.

Ex: Plot the points with polar coordinates:

$A = (2, \frac{\pi}{4})$, $B = (1, \frac{\pi}{6})$, $C = (3, -\frac{3\pi}{4})$

We can extend the definition of polar coordinates to include negative r values too:

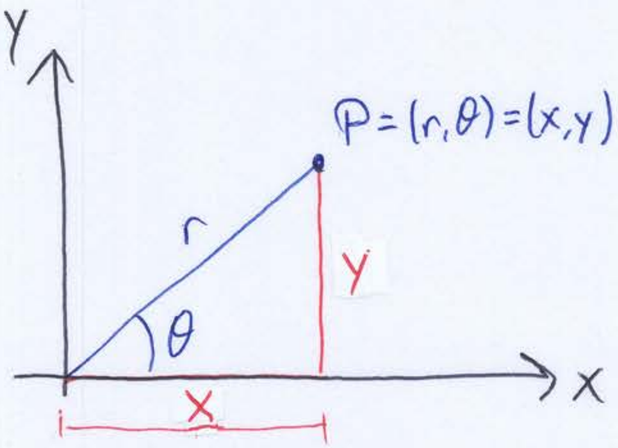
$$(-r, \theta) := (r, \theta + \pi)$$

So, if r is negative, the point (r, θ) is a distance $|r|$ from the origin, with angle $\theta + \pi$

Ex: Plot $D = (-1, \frac{\pi}{3})$ above.

Polar $\longrightarrow$ Cartesian

$x =$ $\qquad\qquad$ $y =$



Ex
Convert $(3, -\frac{\pi}{3})$ to Cartesian.

Cartesian $\longrightarrow$ Polar

Using the above, we can find equations for r & θ in terms of x & y:

Ex: Convert $(1, \sqrt{3})$ and $(-\sqrt{2}, \sqrt{2})$ to polar coordinates.

Graphing In Polar Coordinates

Ex: Graph $r = 3$ & $\theta = \pi/4$

<u>Ex</u>: Graph the equation $r = 3\cos\theta$.
What is an equation for this in Cartesian coordinates?

Polar equations can create some very interesting graphs:

Ex: Sketch the curve

$$r = \cos 3\theta$$

Ex: Sketch the curve

$$r = 1 + \sin \theta$$

Tangents to Polar Curves

If we have a curve $r = f(\theta)$ in polar coordinates, we can create parametric equations out of it:

$$x = r\cos\theta = f(\theta)\cos\theta$$

$$y = r\sin\theta = f(\theta)\sin\theta$$

So, we can find

$$\frac{dy}{dx} =$$

Ex: Consider the curve $r = 3\cos\theta$.

ⓐ Where are vertical tangents?

ⓑ Where are horizontal tangents?

ⓒ Find the tangent line at $\theta = \pi/3$